Valuing bets and hedges: Implications for the construct of risk preference
Shane Frederick, Amanda Levis, Steve Malliaris, Andrew Meyer

Appendices A-E
Appendix A: Discussion of wealth effects

Someone who would only pay $3 for a $10 Heads voucher should pay up to $7 for the Tails voucher – simply because that is the difference between the $10 the pair of vouchers guarantees and their stated valuation for the Heads voucher ($3). This logic assumes that person has purchased the bet at the highest price they would pay (so that owners of a bet are not wealthier than non-owners) or that the stakes are small (so that wealth effects can be neglected). If neither of these is satisfied, this theoretical requirement no longer holds. While the voucher pair is always worth the $10 it guarantees, the division of that $10 between a value of $x for the bet and $(10-x) for its hedge depends on (and indeed is one measure of) the buyer’s risk tolerance. If valuations for each element of the pair are elicited at meaningfully different wealth levels (and hence at different risk tolerances), they need not sum to the prize.

For instance, suppose someone with 2 million dollars and a log utility function for wealth (U=ln(w)) is contemplating purchasing a 10 million dollar coin flip voucher. Such a person would pay about 1.6 million for a Heads voucher and about 7.0 million for Tails voucher (if they had already been given a Heads voucher). Obviously, 1.6 + 7.0 do not sum to 10. Moreover, note the valuation of the bet deviates further from its expected value (3.4 million below) than does the valuation of the hedge (2 million above).

For small stakes contexts, bet and hedge valuations should deviate from EV by a similar (and tiny) amount. The requirement for symmetric deviations is actually easier to appreciate than the requirement that the magnitude of such deviations should be epsilon. The first requires only that one can subtract 3 from 10 to get 7 and notice that 3 and 7 are equidistant from 5. By contrast, EUT’s requirement that small stake risky assets be valued at their expected value requires a recognition that the utility function is continuously differentiable and the amounts involved span a negligible region of relevant wealth levels.

Since EUT doesn’t even permit the emergence of the phenomenon we are investigating, one could instead attempt to view it through the lens of Prospect Theory, which does permit risk aversion “in the small.” Purchasing a bet presumably carries the risk of a “loss” (a wealth reduction that will remain uncompensated if Tails obtains). Correspondingly, one could expect loss aversion to drive down bet valuations substantially below expected value. However, if respondents who already own a bet view subsequent realizations as occurring entirely in the domain of gains, as they would when treating the bet as “house money” (Thaler & Johnson 1990), they cannot “lose” money by purchasing a Tails voucher at any price smaller than the prize. For instance, even if they paid $9 for a Tails voucher, they’d still enjoy a $1 gain. To the extent that hedges are not viewed as instruments to prevent losses, but rather instruments to exchange a larger uncertain gain for a smaller certain gain, they would be worth more than their expected value to the extent that the gains branch of a prospect theoretic value function has appreciable curvature near the reference point. On this view, we’d not expect this positive deviation from EV to be as dramatic as the bet’s negative departure from it, because the curvature of the utility function in gains is less pronounced than the kink at the reference point separating gains from losses, which exhibits a discontinuous change in slope.

Note that the small stakes curvature assumed in Prospect Theory (coupled with some additional assumptions about how respondents holding a risky asset encode the realization of subsequent wealth
levels) yields much the same result as EU with large stakes: bet and hedge valuations that sum to less than the prize, and bet valuations that fall further below EV than hedge valuations rise above it. This prediction is compatible with some aspects of our data, but not all; it can explain bet and hedge values summing to less than the certain prize, but cannot explain their positive correlation.

To see why, consider the valuations of a Prospect Theory agent who (i) buys a bet, and then (ii) buys a hedge, while treating the bet as a pure gain (i.e., as offering either a gain of 10 or a “loss” of 0, relative to their reference point). Formally, the agent’s willingness to pay for a bet is \( x = 10 \left[ 1 + \lambda^{(1/\alpha)} \right]^{-1} \), which follows from rearrangement of her indifference condition, \( v(0) = \frac{1}{2} v(10 - x) + \frac{1}{2} v(-x) \). If after acquiring her bet, she treats its upside entirely as a gain, her willingness to pay for a hedge will be \( y = 10 \left( 1 - \left( \frac{1}{2} \right)^{(1/\alpha)} \right) \). Considering parameter values in the usual range (\(-\lambda > 1; 0 < \alpha \leq 1\)), this willingness to pay is at its minimum when \( \alpha = 1 \), when it equals 5. As her value function’s curvature increases (i.e., as \( \alpha \) falls from one to zero), one can see that bet valuations fall, while hedge valuations rise, generating a negative correlation.

As with EUT, Prospect Theory also requires the bet and hedge prices to sum to the prize if the person paid their full value for the bet. Then, she will value the hedge at the price, \( y \), that solves \( 0 = \frac{1}{2} v(10 - x) + \frac{1}{2} v(-x) = v(10 - x - y) \), where the first equality holds because her willingness to pay is \( x \). Rearranging generates a hedge price of \( y = 10 - x \) which, of course, also covaries negatively with the bet price \( x \).
Appendix B: Experiment materials

Experiment 1a (73 participants began the study; 0 dropped; final sample = 73)

Research assistant script:
This is a “Tails Voucher.” It is worth $10 if a coin flip lands tails, and nothing if it lands heads.

In this experiment, I’m going to ask you how much you would pay for this Tails voucher in four different situations.

Here is $10 in monopoly money (10 one dollar bills). And here is a coin that I will flip one time.

I’m going to ask you to tell me the most that you would be willing to pay for the tails voucher. Then, I’m going to draw a number between 1 and 10. It will represent our price for the voucher. If your willingness to pay is lower than our price, you won’t get the voucher, and you’ll keep your money. If your willingness to pay is higher than our price, you’ll pay our price. So, the higher your willingness to pay, the more likely you are to pay money and get the voucher. But most times that you do get the voucher, you’ll pay less than your max willingness to pay. In short, it is always in your best interest to give your true maximum willingness to pay, because the price is determined by the number we draw, not the number you say.

What is the most you would be willing to pay for the Tails voucher? Place that amount of money in the middle of the table.

[Record the amount under “hyp,” “bet buy,” “wtp.” Draw number with F9 command. Record the number under “hyp,” “bet buy,” “our price.”]

Our price for the voucher is [our price].

[If the random number is higher than their WTP...]

Because your willingness to pay is lower than our price, you keep your money and don’t get a voucher.

[Flip coin.]

[If Tails]

It came up tails. You would have gotten $10 if you bought the voucher.

[If Heads]

It came up heads. You wouldn’t have gotten anything if you bought the voucher.

[If the random number is lower than their WTP...]

Because your willingness to pay higher than our price, you get the voucher, and we take [price].

[Give them the voucher. Take the amount of money indicated by our price and move it to your pile of cash. Tell them to take back the portion that you didn’t take.]

[Flip coin.]
[If Tails]

It came up tails. Here are $10. I’ll take back the voucher.

[If Heads]

It came up heads. The voucher isn’t worth anything. I’ll take it back.

Ok, now we’re moving on to the second situation. We’re going to do the same thing again. But this time, I’m going to give you a heads voucher before we start. It pays $10 if the coin lands heads. Here is a heads voucher, and another $10 dollars in monopoly money.

Now, what is the most you would be willing to pay for a Tails voucher? Place that amount of money in the middle of the table.

[Record the amount under “hyp,” “hedge buy,” “wtp.” Draw number with F9 command. Record the number under “hyp,” “hedge buy,” “our price.”]

Our price for the voucher is [our price].

[If the random number is higher than their WTP…]

Because your willingness to pay is lower than our price, you keep your money and don’t get a Tails voucher.

[Flip coin.]

[If Tails]

It came up tails. You would have gotten $10 if you bought the voucher. You can give me back your Heads voucher.

[If Heads]

It came up heads. Here are $10. You can give me back your Heads voucher.

[If the random number is lower than their WTP…]

Because your willingness to pay higher than our price, you get the Tails voucher, and we take [price].

[Give them the voucher. Take the amount of money indicated by our price and move it to your pile of cash. Tell them to take back the portion that you didn’t take.]

[Flip coin.]

[If Tails]

It came up tails. Here are $10. I’ll take back both vouchers.

[If Heads]

It came up heads. Here are $10. I’ll take back both vouchers.
Experiment 1b (1223 MTurk participants began the study; 47 dropped\(^1\); final sample = 1176)

Suppose...

1) A "HEADS Voucher" pays out $10 if a coin lands heads.
2) A "TAILS Voucher" pays out $10 if a coin lands tails.
3) A single coin will be flipped once.

If you currently don't have any Vouchers, what is the most you would pay for a HEADS Voucher? $____
If you already have a HEADS Voucher, what is the most you would pay for a TAILS Voucher? $____

\(^1\) Exclusion rule: Participants were dropped if they did not respond to one or both questions, if either of their answers contained non-numeric characters, or if either of their answers were less than zero or greater than the prize.
Experiment 1c (1230 MTurk participants began the study; 244 dropped\textsuperscript{2}; final sample = 986)

In this study, you will imagine flipping a fair coin once. We will describe four different scenarios in which this coin flip could occur.

In each scenario a "HEADS Voucher" pays out $10 if the coin lands heads and a "TAILS Voucher" pays out $10 if the coin lands tails.

Scenario #1 (Buying TAILS)
Suppose a fair coin will be flipped one time and you currently don't own either voucher.

Remember:
A HEADS Voucher pays out $10 if the coin lands heads.
A TAILS Voucher pays out $10 if the coin lands tails.

Would you pay $9.50 for a TAILS Voucher?  
Would you pay $8.50 for a TAILS Voucher?  
Would you pay $7.50 for a TAILS Voucher?  
Would you pay $6.50 for a TAILS Voucher?  
Would you pay $5.50 for a TAILS Voucher?  
Would you pay $4.50 for a TAILS Voucher?  
Would you pay $3.50 for a TAILS Voucher?  
Would you pay $2.50 for a TAILS Voucher?  
Would you pay $1.50 for a TAILS Voucher?  
Would you pay $0.50 for a TAILS Voucher?

\textsuperscript{2} Exclusion rule: Participants were dropped if their willingness to pay for bets or hedges (Scenarios 1 and 2) was unintelligible (that is, if they did not respond to one or more questions in the BDM list, or if their responses contained one or more backwards crossovers (wherein they agreed to a less-favorable transaction but declined a more-favorable one)).
Scenario #2 (Buying TAILS)

Suppose a fair coin will be flipped one time and you already own one HEADS Voucher.

Remember:
A HEADS Voucher pays out $10 if the coin lands heads.
A TAILS Voucher pays out $10 if the coin lands tails.

Would you pay $9.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $8.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $7.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $6.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $5.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $4.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $3.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $2.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $1.50 for a TAILS Voucher?  ○ Yes  ○ No
Would you pay $0.50 for a TAILS Voucher?  ○ Yes  ○ No

Scenario #3 (Selling TAILS)

Suppose a fair coin is going to be flipped one time and you currently own one TAILS Voucher.

Remember:
A HEADS Voucher pays out $10 if the coin lands heads.
A TAILS Voucher pays out $10 if the coin lands tails.

Would you sell your TAILS Voucher for $9.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $8.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $7.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $6.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $5.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $4.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $3.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $2.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $1.50?  ○ Yes  ○ No
Would you sell your TAILS Voucher for $0.50?  ○ Yes  ○ No
Scenario #4 (Selling TAILS)

Suppose a fair coin is going to be flipped one time and you currently own one HEADS Voucher and one TAILS Voucher.

Remember:
A HEADS Voucher pays out $10 if the coin lands heads.
A TAILS Voucher pays out $10 if the coin lands tails.

Would you sell your TAILS Voucher for $9.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $8.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $7.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $6.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $5.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $4.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $3.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $2.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $1.50?  ☐ Yes ☐ No
Would you sell your TAILS Voucher for $0.50?  ☐ Yes ☐ No
Experiment 1d (239 participants began the study; 13 dropped\(^3\); final sample = 226)

In-lab paper survey materials:

Imagine you will have one opportunity to flip a coin. If it lands with a red side facing up, you will receive $10.

Imagine you are given a coin with no red faces.

How much would you pay to have one of its faces painted red, so that you will have a 50% chance of receiving $10? $_____

Imagine you paid that amount to have one face of the coin painted red.

How much would you pay to have the other face painted red, so that you will have a 100% chance of receiving $10? $_____

\(^3\) Exclusion rule: Participants were dropped if they did not respond to one or both questions, if either of their answers contained non-numeric characters, or if either of their answers were less than zero or greater than the prize.
Experiment 1e (807 MTurk participants began the study; 123 dropped⁴; final sample = 684)

Would you pay $9.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $8.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $7.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $6.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $5.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $4.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $3.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $2.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $1.50 for a 50% chance to receive $10?  ◯ Yes ◯ No
Would you pay $0.50 for a 50% chance to receive $10?  ◯ Yes ◯ No

If you currently had a 50% chance to receive $10, would you pay $9.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $8.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $7.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $6.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $5.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $4.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $3.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $2.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $1.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No
If you currently had a 50% chance to receive $10, would you pay $0.50 to upgrade it to a 100% chance to receive $10?  ◯ Yes ◯ No

⁴ Exclusion rule: Participants were dropped if they did not respond to one or more questions, or if their bet or hedge valuations contained one or more backwards crossovers (wherein they agreed to a less-favorable transaction but declined a more-favorable one).
Experiment 1f (248 participants began the study; 41 dropped\(^5\); final sample = 207)

MBA class:
Shane Frederick explained the vouchers and auction procedure to the class, and they recorded their valuations on the following form:

1. What is the most you would be willing to pay for a PATRIOTS voucher?
   I’d pay up to $______, but no more.

2. What is the most you would be willing to pay for a FALCONS voucher?
   I’d pay up to $______, but no more.

3. If you were given a PATRIOTS voucher, what is the most you would be willing to pay for a FALCONS voucher?
   I’d pay up to $______, but no more.

4. If you were given a FALCONS voucher, what is the most you would be willing to pay for a PATRIOTS voucher?
   I’d pay up to $______, but no more.

5. If you were given both vouchers, what is the least you would demand to sell your PATRIOTS voucher?
   I’d sell it for as little as $______, but no less.

6. If you were given both vouchers, what is the least you would demand to sell your FALCONS voucher?
   I’d sell it for as little as $______, but no less.

7. What is the most you’d be willing to pay for a pair of vouchers (one PATRIOTS voucher & one FALCONS voucher)?
   I’d pay up to $______, but no more.

---

\(^5\) Exclusion rule: Our analysis is of the Patriots bet and the Falcons hedge. Participants were dropped if their bet (Q1) or hedge (Q3) valuations contained non-numeric characters, were less than zero or greater than the prize, were unintelligible, or were blank. Analysis of the Falcons bet and Patriots hedge yield nearly identical results.
Instructions:
We will ask you about three scenarios involving $5 and $10 Amazon gift cards.

Suppose a "HEADS Voucher" entitles you to a $10 gift card if a coin flip lands heads.
Suppose a "TAILS Voucher" entitles you to a $10 gift card if a coin flip lands tails.

Please consider each scenario independently.

Scenario 1:
Suppose a coin will be flipped once.

What is the most you would pay for a TAILS Voucher? $ _____
If you already own a TAILS Voucher, what is the most you would pay for a HEADS Voucher on that same flip? $ _____

Scenario 2:
Suppose you're buying $5 gift cards.

What is the most you would pay for a $5 gift card? $ _____
If you already own a $5 gift card, what is the most you would pay for a second $5 gift card? $ _____

Scenario 3:
Suppose a coin will be flipped twice. (Recall: you get a $10 gift card for every winning voucher.)

What is the most you would pay for a TAILS Voucher on the first flip? $ _____
If you already own a TAILS Voucher on the first flip, what is the most you would pay for a TAILS Voucher on the second flip? $ _____

Comprehension check and comprehension check results:
Suppose a coin will be flipped once, and you have both a HEADS voucher and a TAILS voucher on that flip.
How many Amazon dollars would you receive if the coin came up heads? $ _____ 79% correct
How many Amazon dollars would you receive if the coin came up tails? $ _____ 77% correct

Suppose a coin will be flipped twice, and you have a TAILS voucher on both the first flip and the second flip.
How many Amazon dollars would you receive if the coin came up tails both times? $ _____ 68% correct
How many Amazon dollars would you receive if the coin came up tails once and heads once? $ _____ 61% correct

Suppose a coin will be flipped once, and you have a TAILS voucher on that flip.
How many Amazon dollars would you receive if the coin came up heads? $ _____ 77% correct
How many Amazon dollars would you receive if the coin came up tails? $ _____ 61% correct

Suppose a coin will be flipped once, and you don't have any vouchers.
How many Amazon dollars would you receive if the coin came up heads? $ _____ 67% correct
How many Amazon dollars would you receive if the coin came up tails? $ _____ 67% correct

6 Exclusion rule: Participants were dropped if their bet or hedge valuations (Scenario 1) contained non-numeric characters, were less than zero or greater than the prize, or were blank.
Experiment 4 (187 MTurk participants began the study; 5 dropped; final sample = 182) (Note: each question appeared on a separate page; each “hedge” question followed its matched “bet” question; the order of the four pairs was randomized)

What is the most you would pay for a 20% chance to win $100? ___

Suppose you have a 20% chance to win $100. What is the most you would pay to increase your chance of winning to 100%? ___

What is the most you would pay for a 40% chance to win $100? ___

Suppose you have a 40% chance to win $100. What is the most you would pay to increase your chance of winning to 100%? ___

What is the most you would pay for a 60% chance to win $100? ___

Suppose you have a 60% chance to win $100. What is the most you would pay to increase your chance of winning to 100%? ___

What is the most you would pay for an 80% chance to win $100? ___

Suppose you have an 80% chance to win $100. What is the most you would pay to increase your chance of winning to 100%? ___

---

7 Exclusion rule: Participants were dropped if they did not respond to one or more questions, if any of their answers contained non-numeric characters, or if any of their answers were less than zero or greater than the prize.
Appendix C: Relation between responses and Cognitive Reflection Test performance

Pooling across the five studies where the CRT was administered, Figure C1 illustrates the claim that the positive correlation between bet and hedge weakens with CRT score.

![Figure C1: bet and hedge valuations for each individual CRT score](image)

Table C1 takes the overall relationship between CRT and bet-hedge correlation and shows that it holds for each individual study.

<table>
<thead>
<tr>
<th>CRT score</th>
<th>1A (n=73)</th>
<th>1B (n=1176)</th>
<th>1B Sub. (n=267)</th>
<th>1C (n=986)</th>
<th>1D (n=226)</th>
<th>1E (n=684)</th>
<th>1F (n=207)</th>
<th>1G (n=1285)</th>
<th>1G Sub. (n=240)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.69</td>
<td>0.78</td>
<td>0.89</td>
<td>0.70</td>
<td>--</td>
<td>0.34</td>
<td>--</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>1</td>
<td>-0.11</td>
<td>0.71</td>
<td>0.66</td>
<td>0.57</td>
<td>--</td>
<td>0.23</td>
<td>--</td>
<td>0.80</td>
<td>0.94</td>
</tr>
<tr>
<td>2</td>
<td>0.70</td>
<td>0.59</td>
<td>0.58</td>
<td>0.60</td>
<td>--</td>
<td>0.15</td>
<td>--</td>
<td>0.70</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>-0.06</td>
<td>0.40</td>
<td>0.75</td>
<td>0.33</td>
<td>--</td>
<td>0.05</td>
<td>--</td>
<td>0.60</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Test of trend: t = -1.3 t = -8.3 t = -2.0 t = -6.5 -- t = -3.5 -- t = -3.2 t = -2.7
Figure C2 shows average CRT score for each bet and hedge WTP pair. Higher average CRT pairs are greener and lower average CRT pairs are redder. The upper left quartile—in which people pay less than $5 for the bet and more than $5 for the hedge—shows highest CRT scores. The upper right quartile—in which people pay more than $5 for both the bet and hedge—shows lowest CRT scores. CRT scores are typically low when bet = hedge ≠ $5, and they are especially low when bet = hedge = $10 or bet = hedge = $0.

Figure C2: Bet and hedge valuations, colored by mean CRT score (N = 4,711)

Note: coloring by CRT score runs from 0 = red through 1.5 = Yellow to 3 = green. Dot area directly proportional to number of responses.
Table C2 divides responses into five types and shows their frequency among each CRT individual score. The two right-most columns of the table show no increase in the percentage of responses summing to $10 (unless we exclude responses where hedge = bet). Among participants whose bet and hedge valuations fail to sum to $10, higher CRT respondents are more likely to value the hedge above the bet and less likely to value the two assets equally.

<table>
<thead>
<tr>
<th>CRT score</th>
<th>bet + hedge ≠ 10 &amp; ...</th>
<th>bet + hedge = 10 &amp; ...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>hedge &lt; bet</td>
<td>hedge = bet</td>
</tr>
<tr>
<td>0</td>
<td>16%</td>
<td>42%</td>
</tr>
<tr>
<td>1</td>
<td>18%</td>
<td>39%</td>
</tr>
<tr>
<td>2</td>
<td>17%</td>
<td>35%</td>
</tr>
<tr>
<td>3</td>
<td>14%</td>
<td>26%</td>
</tr>
</tbody>
</table>

Test of trend: t = -1.3, t = -8.6, t = 9.1, t = 2.2, t = 0.2

Table C3 shows that, in general, high CRT participants paid less for the bet and more for the hedge.

<table>
<thead>
<tr>
<th>CRT score</th>
<th>Bet WTP (N = 4,749)</th>
<th>Hedge WTP (N = 4,726)</th>
<th>Difference (N = 4,711)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4.25</td>
<td>$4.46</td>
<td>$0.21</td>
</tr>
<tr>
<td>1</td>
<td>$4.17</td>
<td>$4.53</td>
<td>$0.36</td>
</tr>
<tr>
<td>2</td>
<td>$3.87</td>
<td>$4.48</td>
<td>$0.63</td>
</tr>
<tr>
<td>3</td>
<td>$3.77</td>
<td>$5.04</td>
<td>$1.28</td>
</tr>
</tbody>
</table>

Test of trend: t = -4.9, t = 4.3, t = 9.6
### Appendix D: Study 3 numeracy test

<table>
<thead>
<tr>
<th>Solution rates, by item</th>
<th>Hedge valuation</th>
<th>number of responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is 25% of 100?  ____</td>
<td>$3 502</td>
<td>$7 53</td>
</tr>
<tr>
<td>What is 10% of 40?  ____</td>
<td>90%</td>
<td>83%</td>
</tr>
<tr>
<td>Express 3/5 as a decimal  ____</td>
<td>90%</td>
<td>75%</td>
</tr>
<tr>
<td>Express 1/20 as a decimal  ____</td>
<td>70%</td>
<td>57%</td>
</tr>
<tr>
<td>When expressed as a fraction, 1.25 is the same as  ____ / 4</td>
<td>66%</td>
<td>51%</td>
</tr>
<tr>
<td>When expressed as a fraction, 0.875 is the same as  ____ / 8</td>
<td>68%</td>
<td>51%</td>
</tr>
<tr>
<td>Two is  ____ % as big as five.</td>
<td>74%</td>
<td>51%</td>
</tr>
<tr>
<td>Six is  ____% bigger than four.</td>
<td>44%</td>
<td>21%</td>
</tr>
<tr>
<td>Total Score</td>
<td>19%</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td><strong>5.22</strong></td>
<td><strong>3.94</strong></td>
</tr>
</tbody>
</table>
Appendix E: Discussion of valuations under multiple reference points

This appendix explores how Koszegi and Rabin’s (2007) theory of multiple reference points applies to our setting. We will assume that additional reference points are adopted immediately upon acquisition (or anticipated acquisition) of the bet. We will assume agents assign a reference point to each outcome that might obtain, and evaluate departures from those reference points using a piecewise linear Prospect Theory function, with $\lambda = 2$, with various outcomes weighted by the probability of occurrence.

In normal settings, minding one’s own business, and not expecting to transact, we will assume agents with a single reference point (which we will describe as “at zero”; i.e., at her background wealth). Faced with an unexpected opportunity to purchase a 50% chance to win $10, the agent could calculate the price $x$ at which she would just be willing to purchase that bet, as shown below:

\[ v(0) = 0.5 v(10-x) + 0.5 v(-x) \]
\[ 0 = 0.5 (10-x) + 0.5 (\lambda)(-x) \]
\[ x = 10/3 \]

At prices above $x$, she rejects the bet and still has a single reference point at zero. But for prices below $x$, she adopts additional reference points to accommodate alternate realizations of the bet that might obtain. Therefore, to verify that the agent will indeed follow through with her plan to purchase the bet, we need to verify that a bet priced at $x=10/3$ remains attractive given the additional reference point(s) its purchase would generate. It does, as we explain in footnote 8.\(^8\)

Assuming an agent possessing the bet will have one reference point at 10 (if the bet pays off) and one reference point at 0 (if it doesn’t),\(^9\) we can calculate the highest price, $y$, she’d pay for the opportunity to hedge her existing bet, by subtracting the cost of the hedge from the two relevant reference points. As shown below, this yields a hedge valuation equal to its expected value ($5). Furthermore, the hedge price is also in equilibrium, in the sense that it is at least as desirable upon acquisition as it was in contemplation.\(^{10}\)

---

\(^8\) Koszegi and Rabin introduce the concept of *unacclimating personal equilibrium* (UPE) which $x=10/3$ satisfies only if an agent who expects to buy the bet for $x$ will indeed follow through with her plan once her reference points adjust to accommodate various realizations of her forthcoming risky purchase. Note that the equation above characterizes the price below which she is not able to forgo the bet, even if her original plan was to reject it. Once she has formulated the intention of buying the bet for $x=10/3$, her reference points shift; let us call them 0 and 10. Now, she compares her two choices – either buying the bet, or not buying it – to the reference points generated by her expectation to buy it. That is, she verifies that $x$ satisfies:

\[ 0.25 v(10-0) + 0.25 v(10-10) + 0.25 v(0-10) + 0.25 v(0-0) \geq 0.5 v(x-0) + 0.5 v(x-10). \]

The left hand side describes her well-being if she buys the bet expecting to buy it; the right hand side describes her well-being if she doesn’t buy the bet while expecting to buy it (thus her wealth is $+x$, representing her foregone expenditure, relative to reference points of 10 and 0. Indeed, the right hand side is smaller; having formulated a plan to buy the bet, she will execute it. Buying the bet at $x=10/3$ is a UPE.

\(^9\) Note that we could equivalently describe these reference points as being at “10-x” and “-x”, as long as we appropriately translate final wealth levels in each possible state: in that case, final wealth would be $(10-x)$ if the bet paid off, $(x)$ if it did not, and $(10-x-y)$ if the hedge was purchased for $y$. Since $(-x + x)$ is present in each term, it drops out, as would any uniform translation of outcomes and reference points.

\(^{10}\) To verify that $y=5$ is a UPE: Suppose an agent expects to buy the hedge at $5. Then she has a single reference point (call it 0); if she follows through, she enjoys $v(0)$. If she defects, then relative to her reference point, she either (i) gains $5 (if the retained
\[
0.25 v(0-0) + 0.25 v(10-0) + 0.25 v(0-10) + 0.25 v(10-10) = 0.5 v((10-y) - 10) + 0.5 v((10-y) - 0)
\]
\[
0.25 (10) - 0.25 (\lambda) (10) = 0.5 (\lambda) (-y) + 0.5 (10-y)
\]
\[
2.5 - 5 = -y + 0.5 (10-y)
\]
\[
-2.5 = -y + 5 - 0.5y
\]
\[
1.5y = 7.5
\]
\[
y = 5
\]

This price is \textit{insensitive} to \(\lambda\), because the hedged outcome is evaluated against reference points at both 10 (compared to which it converts a \textit{probabilistic} $10 loss into a \textit{certain} $5 loss) and 0 (compared to which it converts a complementary probabilistic \textit{gain} of $10 into a certain gain of $5); in no case do losses become gains, or vice versa. Note that valuations for the bet and the hedge in the multiple reference point world are consistent with our experimental results in the sense that the two prices fail to sum to 10, as EUT (and logic) dictate. However, this account does not generate a positive relation between bet and hedge valuations that we nearly always observe.

The strongest reason to doubt the explanatory power of multiple reference points in our data involves participants’ compensation demanded to \textit{sell} a hedge. Someone endowed with both vouchers would be guaranteed the $10 prize, and thus would have a \textit{single} reference point at $10. Therefore her compensation demanded to sell would be given by

\[
v(10 - 10) = 0.5 v(y + 10 - 10) + 0.5 v(y - 10)
\]
\[
0 = 0.5 y + 0.5 \lambda (y-10)
\]
\[
0 = 1.5 y - 10
\]
\[
y = 10/1.5 = 20/3,
\]

which is indeed also a UPE.\textsuperscript{11} If the behavior we explore was a consequence of participants’ use of multiple reference points, then we would expect to observe \textit{normative} behavior in cases where participants’ reference points collapse to a single point. For example, when comparing \textit{willingness to pay} for bets and \textit{compensation demanded} for hedges, we should observe the normative \(y = 10-x\); we do not. Comparing these prices in Studies 1c and 1e, where we have both willingness to pay for the bet and compensation demanded for the hedge, we find correlations of -0.03 and +0.38, respectively, as shown in the plots below.

\textsuperscript{11} To verify that \(y=20/3\) is a UPE: Once the reference points have adjusted, following through gives \(E[v] = -2.50\). Reneging delivers a guaranteed wealth of -20/3 relative to reference points -10 and 0; that is, an equally likely \textit{gain} of 10/3 or a \textit{loss} of 20/3. Therefore, with \(\lambda=2\), the agent prefers not to defect (-2.5 > -3.3); \(y=20/3\) is again a UPE.
Multiplicity and uniqueness

The above considered the case of an agent who does not expect to transact, faced with a sudden opportunity to buy a bet for less than $10/3 or a hedge for less than $5, or with an opportunity to sell her hedge for more than $20/3. We showed she will indeed change her plans and transact. However, at any less favorable prices, she would not be forced to change her plans. Therefore, it is reasonable that she would report answers of ($10/3, $5, $20/3), respectively, in response to being asked about the most she would pay for a bet, most she would pay for a hedge, and least she would accept to sell a hedge.

A second possibility, over regions where both transacting and not-transacting are in equilibrium, appeals to Koszegi and Rabin’s concept of preferred personal equilibrium. K&R handle instances of multiplicity by proposing that from among the set of permissible (equilibrium) plans, people select the plan which, when evaluated relative to the reference points it generates, leaves them the most well off. Under this approach, however, the unique PPE prices in fact obey the normative relation, \( y = 10 - x \).

---

12 To see an example of where this may occur, consider an agent who is contemplating the purchase of a hedge for \( y = 6 \). Relative to the reference points of not buying it, such a price is unattractive (only a price less than or equal to 5 will be tempting enough, as discussed in the text above), but relative to a reference point of buying the hedge, $6 will be a very attractive price (because reneging on that purchase will re-expose you to risk you thought you had eliminated). Formally, \( \frac{1}{2} \cdot v(5) + \frac{1}{2} \cdot v(6-10) < v(0) \), where the left hand side describes the effects of reneging (one either enjoys a $6 windfall, if the remaining bet pays off, or suffers a $4 loss, if it does not), and the right hand side describes the effects of following through (and where one’s reference point upon buying the hedge is unique, and is always achieved – hence \( v(0) \)).
More specifically, for all bet and hedge prices in \([0,10]\), the unique PPE\(^{13}\) is:

(i) \textit{Buy the bet}: if and only if \(x < \frac{10}{3}\).
(ii) \textit{Sell the bet}: if and only if \(x \geq \frac{10}{3}\).
(iii) \textit{Buy the hedge}: if and only if \(y \leq \frac{20}{3}\).
(iv) \textit{Sell the hedge}: if and only if \(y > \frac{20}{3}\).

If respondents are using the PPE concept rather than the UPE described above – that is, if they \textit{anticipate} the possibility of transaction rather than being \textit{surprised} by it – then their willingness to pay for bets and hedges would revert towards the normative \(y = 10 - x\).

Lastly, for completeness, note that Koszegi and Rabin also propose a final equilibrium concept, \textit{choice-acclimating personal equilibrium} (CPE). In CPE, a fully sophisticated agent may wish to commit to an action she would later prefer \textit{not} to follow through on. Note however, as KR discuss, CPE is primarily appropriate in situations where the choice outcome isn’t realized until long after the initial choice.

**Discussion**

We consider two interpretations of Koszegi and Rabin’s multiple reference point theory. Under one interpretation, agents would buy bets at \(x = \frac{10}{3}\), buy hedges at \(y = 5\), and sell hedges at \(y = \frac{20}{3}\). Under the second interpretation, agents would buy bets at \(x = \frac{10}{3}\), buy hedges at \(y = \frac{20}{3}\), and sell hedges at \(y = \frac{20}{3}\). We believe that their theory might account for the depressed willingness to pay for hedges, but not the positive correlation between bet and hedge valuations, nor the comparatively low selling price of hedges given the low buying price of bets.

Because reference points adjust to planned expenditures in their model, Koszegi and Rabin propose evaluating utility as a linear combination of a reference-dependent portion and a traditional consumption utility. A consumption utility term would be approximately linear over gambles of the magnitudes we consider, so taking a combination of gain-loss utility and linear consumption utility would simply shift the numbers above towards their expected value, \(\$5\). Cross-sectional variation in the weight on consumption utility in the population would generate a negative correlation between bet and hedge valuations, as would cross sectional variation in the loss aversion coefficient \(\lambda\), so it is unlikely that the use of multiple reference points would generate a positive correlation between bets and hedges. Lastly, whether one uses the surprise/UPE equilibrium concept or the PPE equilibrium concept, purchases of bets and sales of hedges should obey the normative relationship.

---

\(^{13}\) Again, to see why this is, first note that an agent who \textit{owns} a bet expects
\[
v(\text{bet} | \text{bet}) = 0.25 v(0-0) + 0.25 v(10-0) + 0.25 v(0-10) + 0.25 v(10-10)\]

This actually evaluates to a \textit{negative} number, which makes sense when one observes that a bet is always risky \textit{relative to itself as a reference point}; the possibility of a \$10 payoff is exciting relative to the reference point of \$0, but the possibility of a \$0 payoff relative to the exciting \$10 win is always more negative than the first is positive. However, owning the bet and hedge \textit{together} is exactly as pleasant as owning neither: \(v(\text{bet+hedge} | \text{bet+hedge}) = v(\text{neither} | \text{neither}) = v(0)\). Because the safe position is more pleasant than the risky one, an agent would always prefer to be fully hedged. Finally, following the earlier analysis above, an agent \textit{doesn’t} have a choice only if \(x < \frac{10}{3}\), or if \(y > \frac{20}{3}\).